and Robinson in the energy range 48.2–168.2 MeV. A fit to these data was also sought employing the "wine-bottle" distribution described in Sec. II of this paper.

Preliminary calculations showed that the cross sections, particularly at the low energies, are more sensitive to changes in the parameter $c_0$ than to changes in $t$ and $w$. Therefore, considering the large discrepancy between experiment and theory at the low energies, only changes in $c_0$ were considered.

It was found that no fit to the low-energy positron data could be found by varying $c_0$. This is a result of the positron's being influenced by the whole nuclear charge with negligible screening at low energy.[1] The parameter $c_0$ would have to be increased by at least 50% to bring the cross sections at $\theta=70°$ into agreement with the data, but this change would destroy the good agreement at the higher energies and conflicts with presently accepted values for the nuclear radius.[8,9,13,14]

A somewhat less drastic change of $c_0$ is called for by the low-energy electron-scattering data. Increasing $c_0$ from 1.097 to 1.207 results in the shift in the curves from "A" to "B" as shown in Fig. 9. This change is in the right direction at low energies but causes disagreement at the higher energies and is sharply at variance with the result[9,11] $c_0=(1.097\pm2)\%$ for the Woods-Saxon distribution. Similar results were obtained when $c_0$ was varied in the "wine-bottle" distribution.

## V. SUMMARY AND CONCLUSIONS

From the comparison of the WS and WB cross sections at various energies, it is found that the difference between the two cross sections reaches a maximum of about 30% at incident momenta of $\sim$100 MeV/$c$, provided that momentum transfers of more than 1.5 $F^{-1}$ are not included in the comparisons.

A calculation of electron and positron scattering cross sections employing the currently accepted static, spherically symmetric nuclear charge distribution disagrees by as much as a factor of 2 with the experimental values of Miller and Robinson in the 50–70-MeV energy range while good agreement is obtained in the 85–170-MeV energy range. It is concluded from the above considerations that additional measurements of electron or positron cross sections at energies between 50 and 180 MeV are desirable not only in order to clarify the discrepancy with the results of Miller and Robinson, but also because comparison with theory at various energies may yield information on the accuracy of the assumptions on which the calculations are based.

## ACKNOWLEDGMENTS

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# Neutron Binding Energies in Heavy Nuclei

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Neutron binding energies in 53 nuclei of mass 81–209 were determined with 15 keV accuracy by measurements of $Q$ values for $(d,p)$ and $(d,t)$ reactions. In seven cases, there are large discrepancies with previously accepted values; these are discussed in detail. There are strong indications of subshell closure at 56 neutrons in zirconium and at 64 neutrons in tin, but these indications are much weaker at 56 neutrons in molybdenum, and nonexistent at 64 neutrons in cadmium.

## INTRODUCTION

NUCLEON binding energies have played an important part historically in nuclear structure physics, and their importance has hardly diminished up to the present. Only with the most recent developments in shell-model calculational techniques has it become possible to make reasonably accurate predictions of them,[1,2] and the interest in these calculations continues at a high level. The situation as regards experimental determinations of nucleon binding energies has been very satisfactory in the mass region $A\lesssim70$ for some time, but until a year or two ago, measurements in heavier mass regions carried rather large errors. It was in order to improve this situation that the work herein described was undertaken.

In this paper, we report on determinations of neutron

[1] L. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 32, No. 9 (1960). R. Arvieu, E. Baranger, M. Veneroni, M. Barnager, and J. V. Gillet, Phys. Letters 4, 119 (1963).

[2] I. Talmi, Rev. Mod. Phys. 34, 704 (1962).

binding energies in heavy nuclei from measurements of $Q$ values for $(d,p)$ and $(d,t)$ reactions. Since this work was begun, accurate mass measurements have become available in much of this region.[3,4] However, in most cases, these give determinations of neutron binding energies only with the help of measurements of total beta-gamma energies which are not always certain. Our method gives a direct and completely independent determination of the neutron binding energy, and in some cases gross errors were uncovered. In addition, our method allows determinations to be made in many cases where there is not sufficient data for an accurate determination by the mass-decay energy method.

## EXPERIMENTAL

The 15-MeV deuteron beam from the University of Pittsburgh cyclotron was used to induce $(d,p)$ and $(d,t)$ reactions in thin targets of the various elements under study. Both the incident deuteron beam and the emerging protons and tritons were magnetically analyzed, and the latter were detected by the tracks they leave in photographic emulsions. The method has been described in detail previously.[5] The reaction angles used in practically all cases were 12° for $(d,p)$ reactions and 30° for $(d,t)$ reactions.

Absolute $Q$ values for these reactions were obtained by comparing the energies for their ground-state transitions with energies of various proton and triton groups from the $Fe^{57}(d,p)$ and $Fe^{57}(d,t)$ reactions, for which the $Q$ values are well known. The photographic plates can be located at three different vertical positions, allowing three spectra to be recorded on the same plate. The top and bottom positions were used for unknowns, while the middle position was used for an $Fe^{57}$ spectrum. The three runs were made consecutively without changing anything but the targets and the plate position. If any complications arose in cyclotron operation before the series was completed, the runs were discarded.

In making comparisons between the unknown and the $Fe^{57}$ standard, two or more lines from the $Fe^{57}$ reaction close to and on both sides of the standard were used. Measurements based on the various $Fe^{57}$ lines generally agreed within less than 10 keV, although in a few scattered cases, larger discrepancies occurred. These differences were compromised giving some added weight to lines closer to the unknown. In a few cases of $(d,t)$ reactions, the $Q$ values for the unknown are higher than for the $Fe^{57}(d,t)$ reaction, so that an extrapolation was necessary. In these cases, larger errors are assigned.

TABLE I. Sources of uncertainty and typical errors expected from them. Values given assume two independent experimental determinations. See detailed discussion in text.

| Source of uncertainty | Typical errors (keV) $(d,p)$ | $(d,t)$ |
|---|---|---|
| 1. Standard $Q$ values | 10 | 10 |
| 2. Target thickness | 8 | 14 |
| 3. Change in cyclotron energy between unknown and standard | 10 | 10 |
| 4. Plate handling | 8 | 5 |
| 5. Spectrograph calibration | 4 | 4 |
| 6. Uncertainty in angle of detection | 1 | 4 |
| 7. Uncertainty in determination of peak center | 6 | 4 |
| 8. Uncertainty in cyclotron energy | 1 | 2 |
| Incoherent sum | 20 | 22 |

The energies were corrected for energy loss in the targets and the recoil energy of the residual nucleus. The energy loss corrections were never more than 80 keV in the $(d,p)$ reactions and 140 keV in the $(d,t)$ reactions, except in two cases where it was some 30% larger; these are assigned larger errors. Target thicknesses were determined by weighing and area measurement, and by measurements of alpha-particle energy loss.

Determinations of all $Q$ values were repeated at least twice on separate days. At least one additional measurement was made using less refined techniques[6]; in cases where there were no obvious inaccuracies in those measurements, they were given some weight in arriving at final values.

## ESTIMATION OF ERRORS

A summary of typical errors expected from various sources is shown in Table I. These were estimated as follows:

*1. Standard Q values.* The $Q$ values for the $Fe^{57}(d,p)$ and $(d,t)$ reactions have been assigned uncertainties of 6 and 3.2 keV, respectively.[7] The excitation energies for the excited states that were used in all but a few cases are uncertain by about 8 keV. Thus, the total uncertainty is about 10 keV for the $(d,p)$ and 8.6 keV for the $(d,t)$. However, the direct $Q$-value measurement in the latter case disagrees with the adjusted value by 9.5 keV,[7] an unusually large amount, so that we increase the error from 8.6 to 10 keV. The standard value adopted for $Fe^{57}(d,t)$ is $Q=-1.378$ MeV, which is midway between the directly measured and adjusted values.

*2. Target thickness.* Determinations of target thickness by weight and area measurements, checked for thickness variation by alpha-particle energy-loss measurements, should give an accuracy of about 5%.

[3] R. L. Bishop, R. C. Barber, W. McLatchie, J. D. Macdougall, P. Van Rookhuyzen, and H. E. Duckworth, Can. J. Phys. **41**, 1532 (1963). W. McLatchie, R. C. Barber, R. L. Bishop, H. E. Duckworth, B. G. Hogg, J. D. Macdougall, and P. Van Rookhuyzen, Can. J. Phys. (to be published).

[4] R. R. Ries, R. A. Damerow, and W. H. Johnson, Phys. Rev. **132**, 1662 (1963). R. A. Damerow, R. R. Ries, and W. H. Johnson, *ibid.* **132**, 1673 (1963).

[5] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **126**, 698 (1962).

[6] R. Patell, M.S. thesis, University of Pittsburgh, 1963 (unpublished).

[7] F. Everling, L. A. Konig, and J. H. E. Mattauch, Nucl. Phvs. **25**, 177 (1961).

TABLE II. Changes in cyclotron energy between successive runs on $Fe^{57}$ standard. Two unknowns were run, and plates and targets were changed between successive runs. Cases not separated by horizontal line are successive runs.

| $\Delta E$ (keV) | $\Delta E$ (keV) | $\Delta E$ (keV) |
|---|---|---|
| +14 | −15 | +17 |
| −17 | +12 | —— |
| —— | −14 | −6 |
| +2 | +25 | −7 |
| —— | −12 | −13 |
| +17 | —— | +13 |
| 0 | +20 | —— |
| −17 | —— | |
| —— | | |

Mean absolute value = 13 keV

However, unusual distributions of beam on the target, localized thickness variations (the alpha-particle energy-loss measurements average over $\frac{1}{8}$-in.-diam circular areas), wrinkles, etc., can contribute additional errors, so that 10% accuracy is assumed here.

*3. Change in cyclotron energy between unknown and standard.* Cyclotron energies are notoriously variable due to thermally induced warping of the dees and deflector, ion source variations, etc., and there is no direct way of controlling or even measuring these variations (in the range of 25 keV or less). This could easily lead to large errors if an energy change occurs between running the unknowns and the standard. The only recourse is to try to keep operating conditions as identical as possible during the series of three runs, and to discard runs where this is impossible. In order to estimate the magnitude of this effect, a study was made of energy variations between successive plates run under the same conditions by observing variations of the positions of $Fe^{57}$ peaks on the middle strips of these plates. The data are shown in Table II. It is seen that the average variation is 13 keV, and variations of the order of 20 keV are not uncommon. Actually, this study can be expected to give unduly pessimistic results. In these tests two unknowns are run and the plate is changed between successive runs, whereas in the data taking procedure, an unknown is always run immediately before or after an $Fe^{57}$ run. Furthermore, this effect is averaged out in making separate runs for determination of the same $Q$ value. Thus, this error is estimated as 10 keV.

*4. Plate handling.* Any deviations from strictly vertical motion in shifting the plate between exposures or in scanning will give a displacement between the $Fe^{57}$ and unknown spectra. Also, errors in reading the position of the markers on the plate make a small contribution. Since the energy dispersion is greater for tritons than for protons, this error is proportionately less in the $(d,t)$ case. Here, as in all sources of error discussed below, the error is reduced by the two or more independent experimental determinations.

*5. Spectrograph calibration.* The calibration of the spectrograph has been determined by the use of known levels, but it is far from perfect. The error due to this depends on the distance between the unknown and the $Fe^{57}$ calibration line. This distance was kept rather small, and the fact that calibration lines on both sides of the unknown were used tends to cancel these effects. They therefore probably do not contribute errors larger than 5 keV except in cases of the high-$Q$-value $(d,t)$

TABLE III. Results of present experiment, expressed as neutron binding energies in MeV.[a] Errors listed represent 80–90% confidence limits.

| | $(d,p)$ | | $(d,t)$ | | Previous | |
|---|---|---|---|---|---|---|
| $Zr^{91}$ | 7.183 | ±0.020 | 7.190 | ±0.020 | 7.199 | ±0.007 |
| $Zr^{92}$ | 8.620 | 0.020 | 8.633 | 0.020 | 8.630 | 0.006 |
| $Zr^{93}$ | 6.718 | 0.020 | | | 6.750 | 0.010 |
| $Zr^{94}$ | | | 8.226 | 0.020 | 8.187 | 0.010 |
| $Zr^{95}$ | 6.448 | 0.020 | | | 6.468 | 0.013 |
| $Zr^{96}$ | | | 7.860 | 0.020 | 7.853 | 0.013 |
| $Zr^{97}$ | 5.563 | 0.025 | | | 5.578 | 0.032 |
| $Nb^{93}$ | | | 8.838 | 0.020 | 8.822 | 0.044 |
| $Nb^{94}$ | 7.190[b] | 0.020 | | | 7.195 | 0.025 |
| $*Mo^{93}$ | 8.077 | 0.020 | | | 7.933 | 0.065 |
| $Mo^{94}$ | | | 9.699 | 0.020 | 9.807 | 0.065 |
| $*Mo^{95}$ | 7.362 | 0.020 | | | 7.371 | 0.005 |
| $Mo^{96}$ | | | 9.181 | 0.020 | 9.155 | 0.005 |
| $*Mo^{97}$ | 6.807 | 0.020 | | | 6.816 | 0.004 |
| $Mo^{98}$ | | | 8.637 | 0.020 | 8.640 | 0.004 |
| $*Mo^{99}$ | 5.912 | 0.020 | | | 6.119 | 0.300 |
| $Mo^{100}$ | | | 8.296 | 0.020 | 8.097 | 0.300 |
| $*Mo^{101}$ | 5.391 | 0.020 | | | | |
| $Pd^{104}$ | | | 10.020 | 0.025 | 10.047 | 0.032 |
| $Pd^{105}$ | 7.092 | 0.020 | 7.108 | 0.030 | 7.063 | 0.016 |
| $Pd^{106}$ | 9.574 | 0.030 | 9.569 | 0.025 | 9.543 | 0.013 |
| $Pd^{107}$ | 6.548 | 0.030 | | | 6.542 | 0.004 |
| $Pd^{108}$ | | | 9.235 | 0.030 | 9.223 | 0.006 |
| $Pd^{109}$ | 6.161 | 0.030 | | | 6.147 | 0.007 |
| $Ag^{109}$ | | | 9.205[c] | 0.030 | 9.182 | 0.021 |
| $Cd^{110}$ | | | 9.916 | 0.025 | 9.857 | 0.006 |
| $Cd^{111}$ | 6.966 | 0.030 | 7.002 | 0.030 | 6.965 | 0.005 |
| $Cd^{112}$ | 9.408 | 0.030 | 9.385 | 0.020 | 9.402 | 0.004 |
| $Cd^{113}$ | 6.543 | 0.025 | 6.571 | 0.030 | 6.534 | 0.004 |
| $Cd^{114}$ | 9.042 | 0.030 | 9.058 | 0.030 | 9.041 | 0.004 |
| $Cd^{115}$ | 6.141 | 0.025 | | | 6.148 | 0.012 |
| $Cd^{116}$ | | | 8.714 | 0.020 | 8.684 | 0.012 |
| $Cd^{117}$ | 5.763 | 0.030 | | | | |
| $In^{115}$ | | | 9.047[d] | 0.030 | 9.022 | 0.013 |
| $In^{116}$ | 6.719[e] | 0.025 | | | 6.759 | 0.061 |
| $Sn^{113}$ | 7.729 | 0.025 | | | 8.041 | 0.012 |
| $Sn^{114}$ | | | 10.310 | 0.020 | 10.005 | 0.012 |
| $Sn^{115}$ | 7.554 | 0.025 | 7.562 | 0.030 | 7.523 | 0.012 |
| $Sn^{116}$ | 9.583 | 0.030 | 9.567 | 0.020 | 9.570 | 0.010 |
| $Sn^{124}$ | | | 8.518 | 0.035 | 8.511 | 0.012 |
| $Sn^{125}$ | 5.755 | 0.030 | | | 5.742 | 0.015 |
| $Au^{197}$ | | | 8.078 | 0.030 | 8.067 | 0.017 |
| $Au^{198}$ | 6.507 | 0.030 | | | 6.497 | 0.008 |
| $Pt^{194}$ | | | 8.384 | 0.020 | | |
| $Pt^{195}$ | 6.133 | 0.020 | 6.118 | 0.020 | 6.178 | 0.040 |
| $Pt^{196}$ | 7.937 | 0.025 | 7.944 | 0.020 | 7.922 | 0.012 |
| $Pt^{197}$ | 5.852 | 0.020 | | | | |
| $Pt^{198}$ | | | 7.563 | 0.020 | | |
| $Pt^{199}$ | 5.572[f] | 0.020 | | | | |
| $Pb^{206}$ | | | 8.096 | 0.030 | 8.124 | 0.035 |
| $Pb^{207}$ | 6.751 | 0.030 | | | 6.731 | 0.008 |
| $Pb^{208}$ | | | 7.372 | 0.025 | 7.376 | 0.008 |
| $Bi^{209}$ | | | 7.474 | 0.030 | 7.432 | 0.010 |

[a] Based on $Fe^{57}(d,t)$ $-Q = -1.373$; $Fe^{57}(d,p)$ $-Q = 7.823$; Except for * based on $Cu(d,p)$.
[b] The ground state was not resolved here but it is expected to be strongly excited, so that the high-energy end of the peak was taken as it.
[c] Assumes 0.081 MeV state is lowest one excited, as expected from theory.
[d] Assumes 0.192 MeV state is lowest excited, as expected from theory.
[e] Assumes 0.069 keV state is lowest excited, as expected from theory.
[f] Ground state is masked by much more strongly excited 30-keV state.

TABLE IV. Discrepancies.

| Case | keV | Case | keV |
|---|---|---|---|
| (a) With McMaster's work (Ref. 9). | | | |
| $Mo^{93}+Mo^{94}$ | 27 | $Cd^{112}+Cd^{113}$ | 37 |
| $Mo^{95}+Mo^{96}$ | 9 | $Cd^{112}+Cd^{113}$ | 14 |
| $Mo^{96}+Mo^{97}$ | 12 | $Cd^{113}+Cd^{114}$ | 18 |
| $Mo^{97}+Mo^{98}$ | 17 | $Cd^{113}+Cd^{114}$ | 6 |
| $Mo^{99}+Mo^{100}$ | 16 | $Cd^{115}+Cd^{116}$ | 11 |
| $Cd^{111}+Cd^{112}$ | 27 | $Pb^{207}+Pb^{208}$ | 8 |
| $Cd^{111}+Cd^{112}$ | 32 | | |
| Average $18.0 \div 1.41 = 12.7$ keV | | | |
| (b) With Minnesota work (Ref. 8). | | | |
| $Zr^{91}$ | 16 | $Cd^{111}$ | 1 |
| $Zr^{91}$ | 9 | $Cd^{111}$ | 37 |
| $Zr^{92}$ | 10 | $Cd^{112}$ | 6 |
| $Zr^{92}$ | 3 | $Cd^{112}$ | 17 |
| $Mo^{95}$ | 9 | $Cd^{113}$ | 9 |
| $Mo^{96}$ | 26 | $Cd^{113}$ | 37 |
| $Mo^{97}$ | 9 | $Cd^{114}$ | 1 |
| $Mo^{98}$ | 3 | $Cd^{114}$ | 17 |
| $Pd^{105}$ | 29 | $Sn^{115}$ | 32 |
| $Pd^{105}$ | 45 | $Sn^{115}$ | 39 |
| $Pd^{106}$ | 31 | $Sn^{116}$ | 13 |
| $Pd^{106}$ | 26 | $Sn^{116}$ | 16 |
| Average 18.3 keV | | | |
| (c) Internal discrepancies. | | | |
| $Zr^{91}$ | 7 | $Cd^{113}$ | 28 |
| $Zr^{92}$ | 13 | $Cd^{114}$ | 16 |
| $Pd^{105}$ | 16 | $Sn^{115}$ | 8 |
| $Pd^{106}$ | 5 | $Sn^{116}$ | 16 |
| $Cd^{111}$ | 36 | $Pt^{195}$ | 15 |
| $Cd^{112}$ | 23 | $Pt^{196}$ | 7 |
| Average = 15.8 keV | | | |

reactions, where extrapolations from the $Fe^{57}$ data are necessary. In these cases, errors up to 25 keV are possible.

*6. Uncertainty in angle of detection.* Due to variations in cyclotron operating conditions, the angle of the incident beam is subject to fluctuations. Studies of this effect have shown that these changes are sometimes as large as 1°. The most important effect of this is in the center-of-mass correction. In $(d,t)$ reactions, this contributes an uncertainty of about 5 keV. In $(d,p)$ reactions, center-of-mass corrections are much smaller because of the lower emitted particle mass and the small angle used (12°), so that the uncertainty from this source is only about 1 keV.

*7. Uncertainty in determination of peak center.* In almost all cases, the center of a peak could be determined to $\frac{1}{2}$ mm on the plate which corresponds to 8 keV in $(d,p)$ and 5 keV in $(d,t)$ reactions. These uncertainties also apply to the $Fe^{57}$ lines, but this effect is reduced because several of these were used in most cases.

*8. Uncertainty in cyclotron energy.* Virtually all errors due to uncertainty in cyclotron energy are compensated for by the use of the $Fe^{57}$ calibration. The largest uncompensated error is in the center-of-mass correction.

Assuming that all the errors discussed here add incoherently, the over-all error calculated in Table I is 20 keV for $(d,p)$ reactions, and 22 keV for $(d,t)$ reactions. In general, these error estimates were on the conservative side, so that it is perhaps ultraconservative to add them. These estimates of over-all errors are therefore probably too large. Other estimates are given in the next section.

## RESULTS AND EVALUATION

The results of this work expressed as neutron binding energies are listed in Table III. They are based on $Q$ values for the $Fe^{57}(d,p)$ and $(d,t)$ reactions of 7.823- and −1.378-MeV, respectively. The cases marked by an asterisk were based on the $Cu^{63}(d,p)$ $Q$ value = 5.691 MeV. Any changes in these standards would change the results by an equal amount. The error estimates given are meant to represent 80–90 percent confidence limits; they were arrived at on the basis of the considerations outlined in the last section plus the number of independent experimental determinations and their agreement with one another. The values listed in the final column of Table III are those given in the recent publications of the Minnesota group[4] in the lighter elements, and from the latest published compilations[8] for the heavier elements.

Some estimate of the accuracy of this work may be obtained by considering cases where determinations of these binding energies are available from direct mass measurements. This is done in Table IV. In Table IV(a), discrepancies with double-neutron binding energy determinations at McMaster University[3] are listed; the average discrepancy is 18 keV. However, these comparisons involve two $Q$-value determinations in the present work, so that this implies an error in the present work of 12.7 keV. In Table IV(b), discrepancies with single-neutron binding energies determined by the Minnesota group[4] are listed. Here, the average discrepancy found is 18.3 keV. When the estimated errors in Ref. 4 are taken into account, this implies an average error in the present work of about 15 keV.

Another check on the accuracy of the present work may be obtained by comparing the binding energy in a given nucleus as determined by $(d,p)$ and $(d,t)$ reactions. This is done in Table IV(c); the average discrepancy here is 15.8 keV. However, this sampling includes an untypical number of cases where an extrapolation from the $Fe^{57}$ calibration was necessary; these cases have been assigned large errors in Table III. Thus, considering the three parts of Table IV, one may estimate the average error in the present determinations to be about 15 keV. This is approximately consistent with *a priori* expectations.

## DISCREPANCIES WITH PREVIOUS WORK

Although the results of this work have somewhat depreciated in value since its inception because of the

[8] L. A. Konig, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. **28**, 1 (1961). A. H. Wapstra, *ibid.* **28**, 29 (1961).

publication of many new measurements, there are still about a dozen cases in Table III where the present work gives a large improvement in the accuracy with which binding energies are known.

However, much more interesting than these new measurements are the seven cases where the present determinations are discrepant with previous ones by amounts well outside the combined stated errors. These cases are $Zr^{93}$, $Zr^{94}$, $Mo^{93}$, $Mo^{94}$, $Cd^{110}$, $Sn^{113}$, and $Sn^{114}$. It may be noted that with one exception, these discrepancies occur in pairs; in all three of these pairs, the

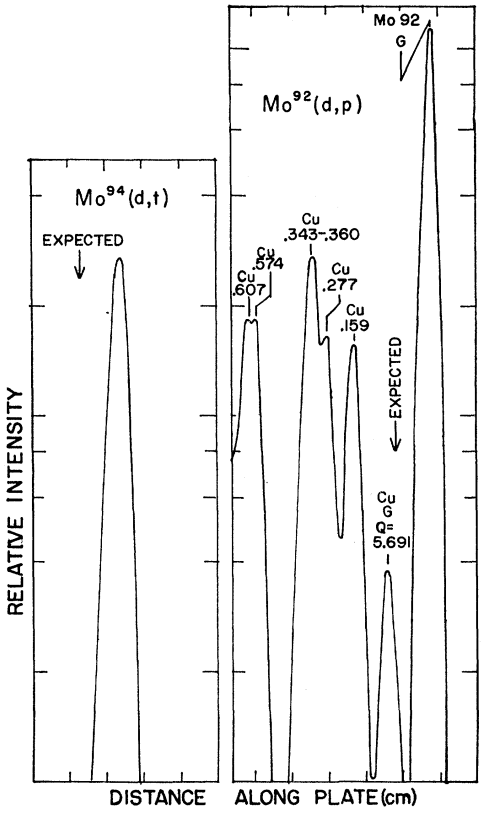


FIG. 1. Energy spectra from $Mo^{94}(d,t)$ and $Mo^{92}(d,p)$ reactions. The targets were homogeneous mixtures of copper and the Mo isotopes. The peak shown in the $(d,t)$ spectrum was the only one in this energy region. The position marked "excited" indicates the peak location expected if the previous values were correct.

discrepancies are such that they can be explained by errors in a single mass; in all cases, these masses were not directly measured, but were determined from beta-gamma decay energies or other indirect methods. The discrepancies would also be explainable if the same excited state of the nucleus in question were excited in both our $(d,p)$ and $(d,t)$ measurements. However, in all cases these regions have been very carefully investigated from the standpoint of nuclear structure.[9–11] There is

[9] B. L. Cohen and O. V. Chubinsky, Phys. Rev. **131**, 2184 (1963).
[10] S. Hjorth and B. L. Cohen (to be published).
[11] E. J. Schneid, A. Prakash, and B. L. Cohen (to be published).

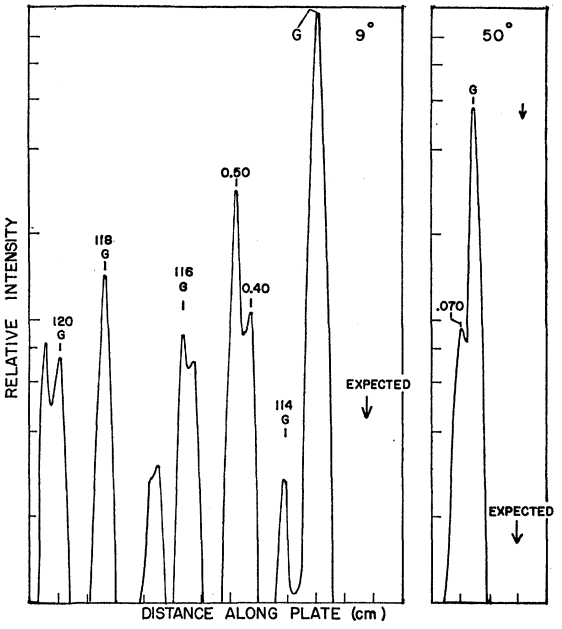


FIG. 2. Energy spectra of protons from $Sn^{112}(d,p)$ reactions. Left figure is at 9°, and right figure is at 50°, where 0.070 MeV state is more clearly in evidence. States labeled *G* were taken as ground states; locations expected from previous values are indicated.

very little possibility that the states observed are not the ground state.

We now discuss the discrepancies individually in detail:

*$Zr^{93}$, $Zr^{94}$.* The sum of the measured binding energies

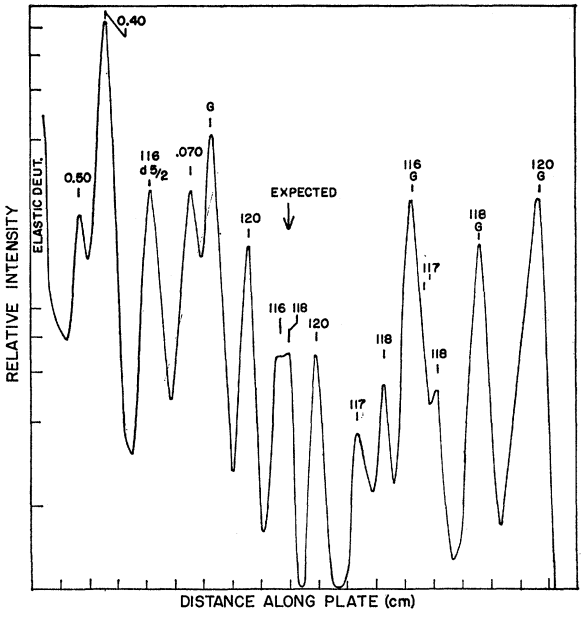


FIG. 3. Energy spectra of tritons from $Sn^{114}(d,t)$ reactions. Target was not very pure isotopically, so that peaks from other isotopes are seen. Note the break in ground-state energies is going from isotopes 120 to 118 to 116 to 114 (labeled "*G*"). Location expected from previous *Q* value is indicated.

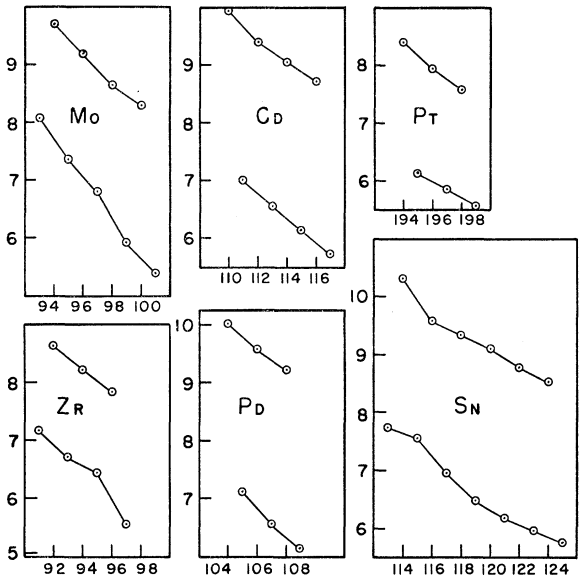


FIG. 4. Neutron binding energy plotted versus $A$. Data are from Table III.

agrees within 7 keV with mass measurements, so that the difficulty is completely explainable by an error in the mass of $Zr^{93}$, as determined from its beta-decay energy to $Nb^{93}$. This region is exceedingly well known from the nuclear structure standpoint,[9] and a level $\sim$35 keV below the strongly excited $d_{5/2}$ state would be very unexpected.

*$Mo^{93}$, $Mo^{94}$*. The experimental energy spectra from the $Mo^{92}(d,p)$ and $Mo^{94}(d,t)$ reactions are shown in Fig. 1. The locations marked "expected" indicate where the peak is expected if the previous determinations were correct. In both cases, there is no room for doubt. There can be no question here of observing an excited state as the observed $Q$ values are larger than those expected from the previous data. The sum of the two binding energies disagrees with the determination from measured masses by 27 keV, far less than the individual discrepancies, which are about 120 keV. The previous determination of the $Mo^{93}$ mass is based on the $Nb^{93}(p,n)$ threshold. That reaction apparently does not excite the ground state.

*$Sn^{113}$, $Sn^{114}$*. Here, the discrepancies are very large; over 300 keV. However, the sum of the two binding energies agrees with the determination from masses within 7 keV. The experimental measurements on $Sn^{112}(d,p)$ and $Sn^{114}(d,t)$ are shown in Figs. 2 and 3. It is clear that the same state is being observed in both cases as the 0.070-, 0.40-, and 0.50-MeV states are observed in both cases. The 0.070-MeV $\frac{7}{2}^+$ state is known from decay scheme work and its spin is known from its measured half-life. This agrees with the determination from the stripping angular distribution. From Fig. 2 it is seen that a state at the position expected from the previous determination must be excited at least 50 times less than is the state we designate as the ground state. Such a state would be completely unexpected from nuclear-structure information, which is abundantly available in this region.[11] The apparent error in the $Sn^{113}$ mass must be due to an error in the beta-gamma decay of $In^{113}$.

*$Cd^{110}$*. The discrepancy here is 59 keV, but this is still far outside the expected error. An excited state known previously is seen; also the nuclear-structure situation is relatively clear in requiring a strongly excited ground state. The previously determined mass of $Cd^{109}$ is based on the electron capture energy of $Ag^{109}$; it seems possible that the error in determining this has been grossly underestimated.

## INDICATIONS OF SHELL CLOSURE

A break in smooth binding-energy dependences may be taken as an indication of shell closure. In order to investigate the possibility of such breaks, data from Table III on various isotopes of a given element are plotted in Fig. 4.

A break at 56 neutrons in Zr, corresponding to the filling of the $d_{5/2}$ subshell, is very clearly seen. Other evidences for this very clear shell closure have been discussed previously.[9] There is a somewhat lesser break at 56 neutrons in the odd isotopes of Mo, but there is no indication of it in the even isotopes. Other evidence[10] indicates that this shell closure is much less definite in Mo than in Zr.

A break at 64 neutrons, indicating closure of the $d_{5/2}+g_{7/2}$ subshells is strongly evident in the even isotopes of Sn. However, curiously, the 65th neutron seems to be extra tightly bound. There is no indication of a 64-neutron shell closure in the Cd isotopes. In the Pd and Pt isotopes, no shell closure is expected and none is found.